Int. J. Heat Mass Transfer. Vol. 8, pp. 511–514. Pergamon Press 1965. Printed in Great Britain

SOLID CARBON-DIOXIDE SUBLIMATION AT AN AXISYMMETRIC STAGNATION POINT

D. B. SPALDING* and F. A. CHRISTIE†
Department of Mechanical Engineering, Imperial College, London, S.W.7

(Received 3 August 1964 and in revised form 30 November 1964)

Abstract—The sublimation rate of solid carbon-dioxide has been measured at an axisymmetric stagnation point in a stream of hot air. The results, are presented as dimensionless conductances covering a driving-force range of $0{\cdot}220 \leqslant B_h \leqslant 0{\cdot}835$.

NOMENCLATURE

B_h, driving force for mass transfer, calculated using enthalpies [1] (equation (1));
g_h, surface conductance for mass transfer (equation (1)), [lb/ft²h];
h, enthalpy (equation (2)), [Btu/lb];
$\dot{m}''$, mass-transfer rate per unit area (equation (2)), [lb/ft²h];
N_{Pr}, Prandtl number;
N_{Le}, Lewis number;
N_{Sc}, Schmidt number;
$\dot{q}_S''$, heat-transfer rate from gas to interface (equation (2)), [Btu/ft²h];
T, temperature, [°R];
u, velocity (equation (1)), [ft/h];
x, streamwise distance dimension (equation (1)), [ft];
μ, absolute viscosity (equation (1)), [lb/ft h];
ρ, density (equation (1)), [lb/ft³].

Subscripts
G, refers to the main-stream state;
GS, a fictitious state having main-stream composition, but interface temperature;
S, refers to the fluid state adjacent to the interface;
sg, denotes sublimation.

* Professor of Heat Transfer.
† Formerly, Research Student; now DSSO 2, Canadian Armament Research and Development Establishment, P.O. Box 1427, Quebec, P.Q., Canada.

THEORY

EVANS [2], Stewart and Prober [3], and others have published solutions of the differential equations which describe the process of mass transfer through a laminar boundary layer, with uniform fluid properties, at an axisymmetric stagnation point. The solutions of Howe and Mersman [4] show that, for air-to-air transpiration, variations in the temperature ratio T_S/T_G have little effect on the conductance over the range $0{\cdot}25 \leqslant T_S/T_G \leqslant 4{\cdot}0$, provided main-stream properties are used to correlate the results. In the tests reported here, the range of temperature ratios was $0{\cdot}265 \leqslant T_S/T_G \leqslant 0{\cdot}580$; we can expect the solutions of [2] and [3] to be valid, when presented in the form:

$$\frac{g_h}{[\mu_G \, \rho_G \, (\mathrm{d}u/\mathrm{d}x)_G]^{1/2}} = f(B_h) \qquad (1)$$

with:

$$B_h \equiv \frac{h_G - h_{GS}}{\dot{q}_S''/\dot{m}''} \qquad (2)$$

and:

$$g_h \equiv \dot{m}''/B_h \qquad (3)$$

The function appearing in equation (1), for an axisymmetric stagnation point with

$$N_{Pr} = 0{\cdot}70,$$

is expressed by the curve in Fig. 2, plotted by reference to the solutions of [2].

APPARATUS

Figure 1 presents the essentials of the test apparatus. A jet of electrically heated air was directed vertically downward against a horizontal stagnation plate, the centre of which was replaced by a disc of solid carbon dioxide, 2 inches in diameter.

The temperature of the air jet varied from 68°F to 762°F, while its velocity was varied from 80 ft/s to 160 ft/s, with a turbulence intensity of 1·5 per cent at the nozzle exit. The disc surface was kept continuously level with the surrounding plate by a manually controlled variable-speed motor; sublimation from the sides and base of the disc was prevented by filling the annuli surrounding the central cylinder with liquid

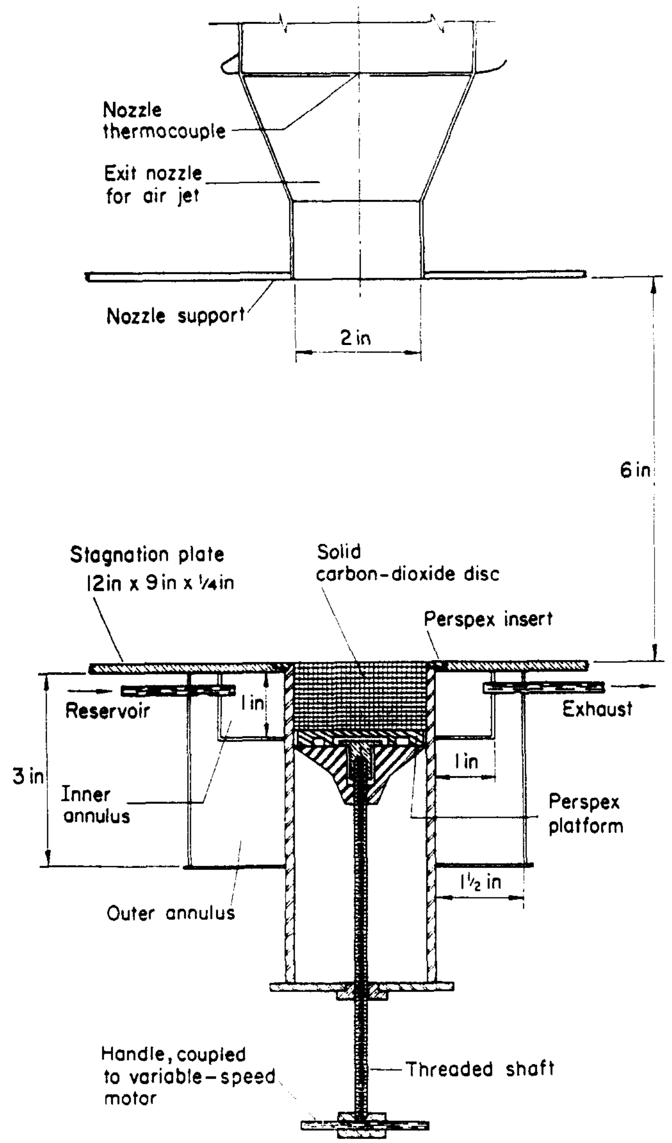

FIG. 1. Test region of apparatus used to obtain sublimation rates of solid carbon-dioxide at an axisymmetric stagnation point.

nitrogen. This allowed direct measurement of the mass lost from the exposed face of the disc during a test, using a Mettler K7 balance.

Any malalignment of the disc surface and the surrounding plate results in a change in the velocity gradient, (du_G/dx). Experiments simulating this condition indicated that the error in (du_G/dx) was 1 per cent or less.

DATA REDUCTION

In addition to the mass-transfer rate, equations (1), (2) and (3) also involve: (i) the u_G variation along the disc radius, (ii) the mean air-stream temperature above the disc, and (iii) the surface temperature of the disc. Preliminary investigations [5] established correlations connecting both (i) and (ii), with (a) the air flow-rate through the nozzle, and (b) the nozzle temperature, measured as shown in Fig. 1. Since $N_{Sc} = 0{\cdot}96$ for carbon-dioxide in air, the Lewis number (N_{Le}) equalled 0·73. Therefore the standard constructions used to calculate T_S, presented in [6] for $N_{Le} = 1$, must be modified by introducing the factor $N_{Le}^{-2/3}$. As explained by [9], for Lewis numbers different from unity, G, S, T are no longer collinear. Introduction of $N_{Le}^{-2/3}$ allows prediction of a G state which is collinear with S, T, permitting graphical solutions. Figure 3 illustrates a $N_{Le} \neq 1$ situation. $\dot{q}_S''$ was taken to equal the enthalpy increase of CO_2 between the initial solid state and the gaseous state at T_S, together with a small radiative correction. The radiative heat flux was estimated using the room and disc-surface temperatures. The heat of sublimation (h_{sg}) was obtained from Perry [7], and density and viscosity values from [8].

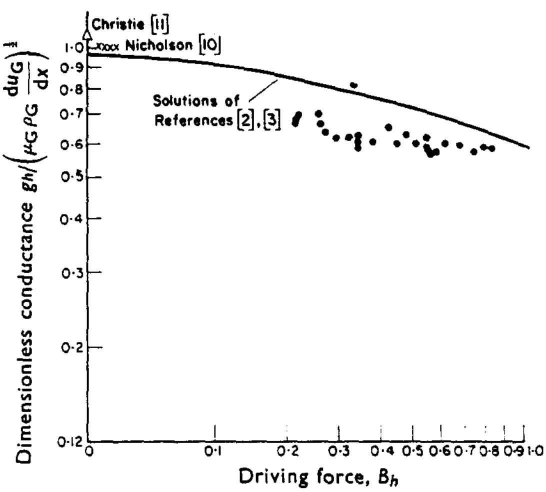

FIG. 2. Comparison of theory with experimental data obtained by sublimation of solid carbon-dioxide at an axisymmetric stagnation point, $N_{Pr} = 0{\cdot}70$.

RESULTS

The results of the experimental investigation are shown in Fig. 2. The calculated results have an r.m.s. scatter of 4·3 per cent about a mean line which lies between 20 per cent (at $B_h = 0{\cdot}220$) and 13 per cent (at $B_h = 0{\cdot}835$) below theory.

Comparison may be made with earlier work

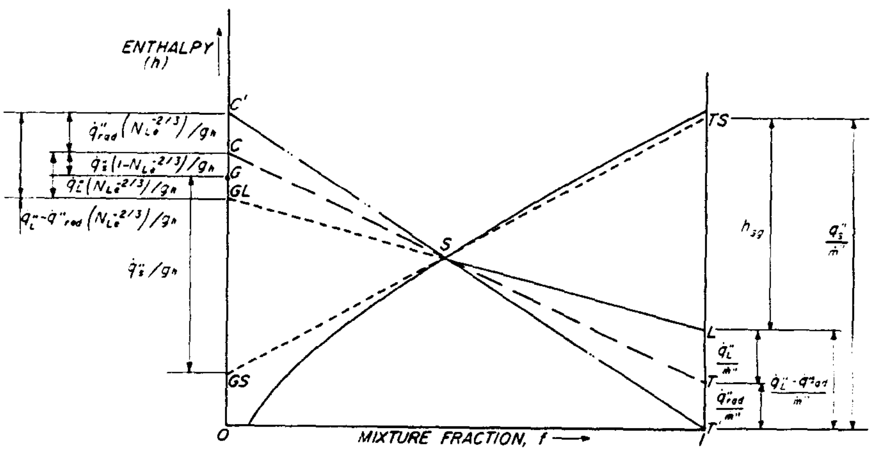

FIG. 3. General construction for solution of the surface temperature (T_S), When $N_{Le} < 1$, $\dot{q}_{rad}'' =$ negative.

carried out using the same apparatus under similar conditions. Nicholson [10] has investigated transpiration of water through a two-inch diameter porous plate, while Christie [11] investigated sublimation from a 2 inch diameter naphthalene disc. The results of these two earlier investigations appear in Fig. 2. They lend support to the theoretically predicted curve.

Possible causes of the disagreement between the present data and theory have been sought, but no final explanation has been found. For instance, experiments with varying air humidity have shown that condensation of water vapour on the disc surface has no significant effect on the mass-transfer rate. Occasional, localized gaps between the solid carbon-dioxide disc and its surrounding cylinder were observed, but these were so limited in extent and irregular in occurrence, that they could account for experimental scatter only.

Finally, both calculation and surface temperature measurement indicate that subcooling of the carbon dioxide by the liquid nitrogen coolant contributes a maximum of 5 per cent (at $B_h = 0{\cdot}220$) to the disagreement between theory and experiment, by causing heat to be conducted away from the carbon-dioxide disc.

CONCLUSION

An apparatus has been described for measuring the rate of sublimation of solid carbon dioxide at an axisymmetric stagnation point in a stream of air. Experimental sublimation rates lie about 16 per cent below theoretical predictions, in the driving force range

$$0{\cdot}220 \leqslant B_h \leqslant 0{\cdot}835.$$

REFERENCES

1. D. B. Spalding, A standard formulation for the steady convective mass-transfer problem, *Int. J. Heat Mass Transfer* **1**, 192 (1960).
2. H. L. Evans, Mass transfer through laminar boundary layers—8. Further solutions to the velocity equation, *Int. J. Heat Mass Transfer* **5**, 373 (1962).
3. W. E. Stewart and R. Prober, Heat transfer and diffusion in wedge flows with rapid mass-transfer, *Int. J. Heat Mass Transfer* **5**, 1149 (1962).
4. J. T. Howe and W. A. Mersman, Solutions of the laminar compressible boundary-layer equation with transpiration which are applicable to the stagnation regions of axisymmetric blunt bodies. *NASA TN*-12 (1959).
5. F. A. Christie, Sublimation of solid carbon-dioxide at a stagnation point, M.Sc. Thesis, University of London (1964).
6. D. B. Spalding, The theory of melting ablation, with vaporization, gas phase chemical reaction, surface pyrolysis, and transient effects, *Aero. Quart.* **12**, 238 (1961).
7. J. H. Perry (Ed.), *Chemical Engineers Hqndbook*, 3rd Ed. McGraw-Hill, New York (1958).
8. J. Hilsenrath *et al.*, *Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon-Dioxide, Carbon-Monoxide, Hydrogen, Nitrogen, Oxygen and Steam*, National Bureau of Standards, Pergamon Press, London (1960).
9. D. B. Spalding, *Convective Mass Transfer—An Introduction*, pp. 133, 135; p. 308 *et seq.* Arnold Press, London (1962).
10. V. Nicholson, Mass-transfer at an axisymmetrical laminar stagnation point. DIC Dissertation, Imperial College, University of London (1961).
11. F. A. Christie, Sublimation at an axisymmetrical laminar stagnation point. DIC Dissertation, Imperial College, University of London (1962).

Résumé—La vitesse de sublimation de la neige carbonique a été mesurée à un point d'arrêt de révolution dans un écoulement d'air chaud. Les résultats son présentés comme des conductances sans dimensions couvrant une gamme de "force motrice" $0{,}220 \leqslant B_h \leqslant 0{,}835$.

Zusammenfassung—In einem heissen Luftstrom wurde an einem achsensymmetrischen Staupunkt die Sublimationsgeschwindigkeit von Kohlendioxyd gemessen. Die Ergebnisse werden als dimensionslose Ableitungen angegeben, die einen Bereich der treibenden Kräfte von $0{,}220 \leqslant B_h \leqslant 0{,}835$ überdecken.

Аннотация—Измерена скорость сублимации твердой двуокиси углерода в осесимметричной критической точке в потоке горячего воздуха. Результаты даны в виде безразмерных значений проводимостей в диапазоне движущей силы $0{,}220 \leqslant B_h \leqslant 0{,}835$.